JOURNAL OF AIRCRAFT
Vol. 46, No. 4, July–August 2009

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Drag Extrapolation to Higher Reynolds Number

Lance W. Traub*
Embry Riddle Aeronautical University,
Prescott, Arizona 86301-3720

DOI: 10.2514/1.43952

Nomenclature

AR = aspect ratio
C_D = drag coefficient
C_{D0} = zero lift drag coefficient
C_L = lift coefficient
i = index
k = slope of linearized drag polar
α = exponent

Subscripts

A, B = lift or drag coefficient at points A and B
$m1$ = maximum lift coefficient at low Re (wind-tunnel data to extrapolate)
$m2$ = maximum lift coefficient at extrapolated Re
1 = low Re (wind-tunnel data to extrapolate)
2 = extrapolation Re

Introduction

EXTRAPOLATION of wind-tunnel data to full scale remains a challenging and demanding undertaking. Although literature is available [1] on the topic, it is often anecdotal and seldom includes methodology to extrapolate. Extrapolation techniques developed over years are seen as proprietary and are seldom published, as they provide significant cost and time advantages over a competitor. Discrepancies between wind-tunnel and flight measurements have many causes: Reynolds number effects, model fidelity, wind-tunnel mount support interference, wall corrections, aeroelastic effects, etc. Additionally, commonly used methods to simulate high Re flow, for example, tripping the boundary layer at the flight estimated location, may also cause errors, as a matching based on shock location or boundary-layer momentum thickness may be more representative depending on the geometry and flight conditions [2].

Current approaches to wind-tunnel data extrapolation show the use of these data in conjunction with computational fluid dynamics (CFD) [2]. High-fidelity wind-tunnel data may be used as an absolute at a particular Re number; CFD is then used to scale the data to flight, as CFD's ability to establish representative changes in drag between Re numbers is well accepted. Simple methods for a quick estimate of drag at a higher Re suitable for educational or preliminary design purposes are limited, whereas some available methods are inexact or sketchy in application [3]. In this Note, a simple method is presented to extrapolate the drag polar of an airfoil or wing to a higher Reynolds number. Following method development, comparisons of the approach with experimental data are presented.

Theoretical Approach

The drag polar for an aircraft or wing section (with limited camber) may be expressed as

$$C_D = C_{D0} + kC_L^2 \tag{1}$$

where C_{D0} is the so-called zero lift drag coefficient (predominantly due to skin friction) and kC_L^2 is the drag due to lift coefficient (due to both vortex and sectional pressure drag for a wing or pressure drag for an airfoil). Extrapolation of a polar represented by Eq. (1) indicates that both drag constituents need to be modified. As C_{D0} is primarily friction based, a common correction to higher Re uses a simple formulation based upon the drag coefficient dependency of a flat turbulent plate with Re. Consequently, the zero lift drag coefficient may be estimated as [2]

$$C_{D02} = C_{D01}(Re_1/Re_2)^{\alpha} \tag{2}$$

Based purely upon the result for a flat plate with turbulent flow, α is set to 0.2. For Re greater than 8 million, a value of 0.11 may provide better estimates [2]. An alternative experimental approach to estimating the C_{D0} at higher Re is to vary the Re within the capability of the wind-tunnel facility so as to formulate a plot of C_{D0} versus Re (using a logarithmic scale for the Re axis). Fitting a linear regression to these data then allows extrapolation to higher Re.

Extrapolation of the lift dependent drag coefficient is facilitated by the observation [3] that the slope of the linearized drag polar (C_D versus C_L^2) has a marginal dependency on Re. Based upon this observation, the following method details a systematic approach to drag polar extrapolation to higher Re. The method entails an extension of the linear portion of the linearized polar to the estimated location of stall onset at higher Re numbers. An estimate of $C_{L\,\max}$ at the Re of extrapolation is required. It is also assumed that the nature or characteristic of the stall is preserved at higher Re when examined using the linearized polar. This results in a more rapid stall progression at higher Re (than in the data extrapolated from) when viewing the traditional drag polar, as is typically seen experimentally. This assumption will be revisited later. Figure 1 defines variables used in the extrapolation procedure, as well as some extrapolation details.

Extrapolation Procedure

The following sequence defines the procedure to extend a given experimental polar to higher Reynolds number. The polar may be that of an airfoil or wing. Note that uppercase symbols, associated with a wing, are used in the presentation. For an airfoil, one may replace C_D with C_d and C_L with C_l. Figure 1 contains details of some procedural steps.

Received 20 February 2009; revision received 12 March 2009; accepted for publication 13 March 2009. Copyright © 2009 by Lance W. Traub. Published by the American Institute of Aeronautics and Astronautics, Inc., with permission. Copies of this paper may be made for personal or internal use, on condition that the copier pay the $10.00 per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923; include the code 0021-8669/09 $10.00 in correspondence with the CCC.

*Associate Professor, Aerospace and Mechanical Engineering Department. Member AIAA.

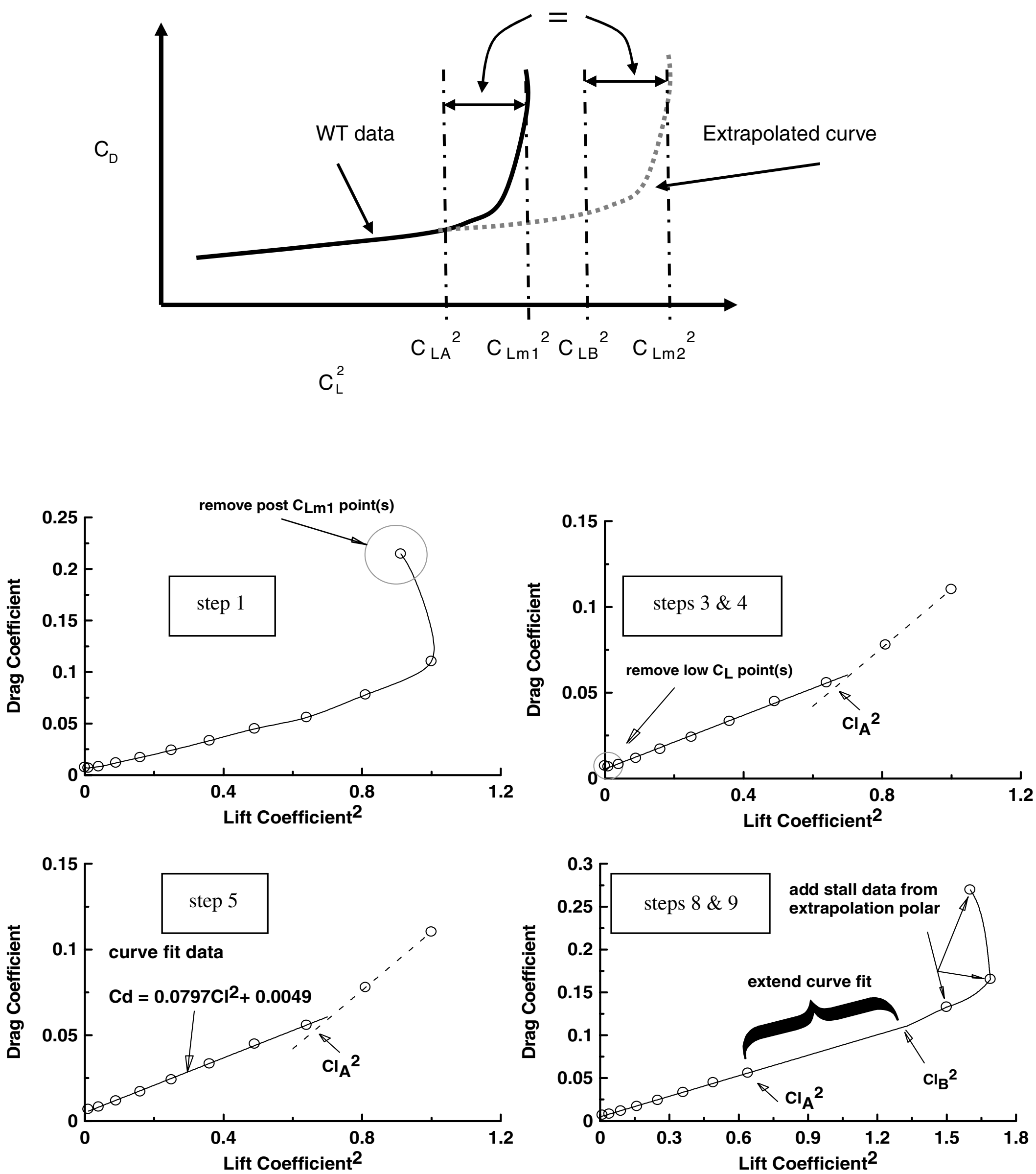

Fig. 1 Variable definitions and extrapolation procedure details.

1) Plot the linearized polar of the wind-tunnel data. Remove post C_{Lm1} points.

2) The polar will be seen to be constituted of generally two linear elements: one associated with attached flow and the other with the initial onset of separation.

3) Fit a linear regression to the plot. Starting from C_{Lm1}, reduce the number of points in the linear regression until a clearly defined break appears (or slope discontinuity) if necessary. This point, defined as C_{LA}^2, is clearly seen in Figs. 2–4.

4) It may also be necessary to eliminate some points from the minimum lift coefficient region to improve the curve fit.

5) Record the curve fit as it will be used to extend the linear region of the polar. This numerates Eq. (1).

6) The next step is to estimate the location at which stall onset occurs at the extrapolated Re.

7) This point, defined as C_{LB}^2 is estimated assuming that the lift coefficient extent (in the linearized polar) over which stall occurs is similar for both the wind-tunnel and extrapolated data. Thus, it is defined as

$$C_{LB}^2 = C_{Lm2}^2 - (C_{Lm1}^2 - C_{LA}^2) \tag{3}$$

8) To extend the polar from C_{LA}^2 to C_{LB}^2:

a) Use the original wind-tunnel data (C_L and C_D) until the point preceding C_{LA}^2.

b) From (and including) points C_{LA}^2 to C_{LB}^2 use the curve fit defined in the preceding point 5 to estimate the drag coefficient.

c) It is necessary to generate incremental lift coefficients (padding) from C_{LA}^2 to C_{LB}^2. The following expression may be used:

$$C_{Li}^2 = C_{LA}^2 + i \cdot (C_{LB}^2 - C_{LA}^2)/N \qquad i \quad \text{from } 0 \quad \text{to } N \tag{4}$$

N is any integer; one may use sufficient points so that C_L increments of approximately 0.05 result.

9) To extend the polar from C_{LB}^2 to C_{Lm2}^2:

a) In this region, the original wind-tunnel data (C_{LA+i}, C_{DA+i}, i.e., the experimental points after the point identified as A in the wind-tunnel linearized polar data) are combined with the extrapolated data. Estimates of the lift and drag coefficient are made using

$$C_{Li+B}^2 = C_{LB}^2 + (C_{LA+i}^2 - C_{LA}^2) \tag{5}$$

$$C_{Di+B} = C_{DB} + (C_{DA+i} - C_{DA}) \tag{6}$$

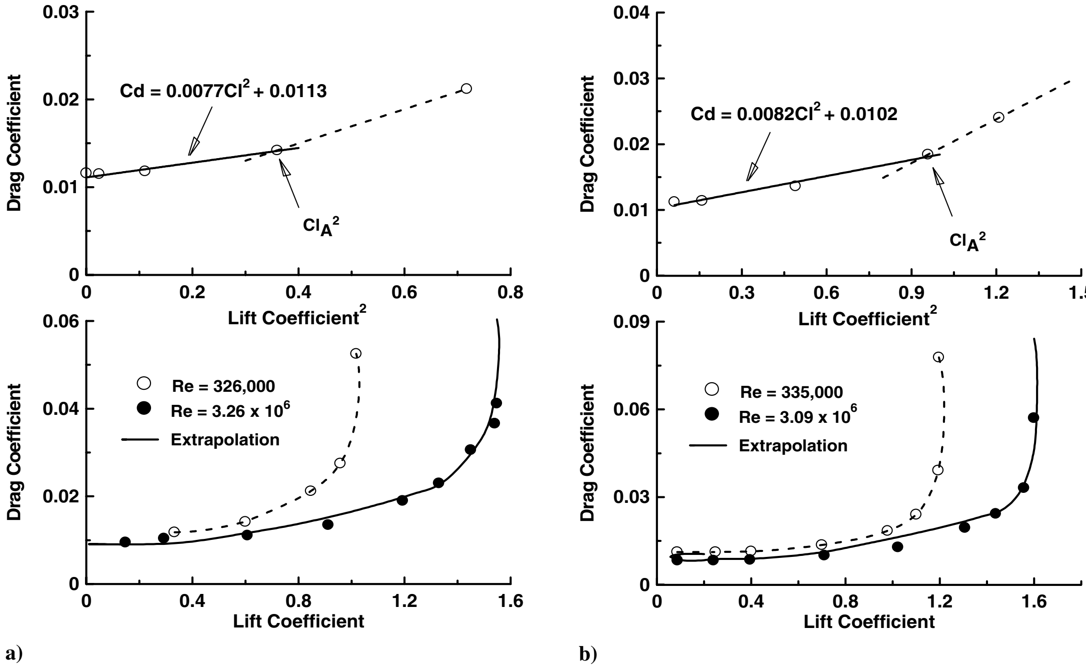

Fig. 2 Comparison of extrapolated polar with experiment, a) NACA 0015 and b) NACA 23012 sections. Data from [5].

The index i is from one to the wind-tunnel data point defined as C_{Lm1}. C_{DB} is from step 8b.

10) The extrapolated polar is then determined by rooting the C_L^2 values.

11) The polar may then be adjusted for C_{D0} effects using Eq. (2). The calculated C_{D0} decrement ($C_{D01} - C_{D02}$) is subtracted from all the C_D values.

12) Cautions: As will be shown, the method is dependent upon two assumptions; that the linearized polar's stall characteristics and attached flow slope [3] are conserved. To ascertain the stall behavior, [4] may be used as a guide. Stall types may vary between long bubble, short bubble, trailing edge, or a combination of short bubble and trailing edge. If a stall type is conserved between the wind-tunnel and extrapolated Re data, then the stall behavior should be reasonably approximated by the wind-tunnel data.

Figures 2–4 show comparisons of wind-tunnel data collected at two Re numbers along with extrapolations of the low Re data to the higher Re. The upper inset of each figure shows the linearized polar identifying the point C_{LA}^2. Figures 2 and 3 contain data from [5] which was collected to determine the effect of Reynolds number on the evaluated airfoil sections. Data for Re of approximately 330,000 as well as 3 million are shown. In [5], incremental Re variations from less than 100,000 to over 3 million are presented. The $Re \approx 330,000$ was chosen for extrapolation as it represents an extreme case, but not one that would be excessively contaminated by low-Re phenomenon. It is also an Re that is readily achieved in most wind-tunnel facilities. The method detailed earlier was used to extrapolate the low-Re data to the higher Re. The high-Re maximum lift coefficient was used in the extrapolation procedure as C_{Lm2}. A value for α of 0.11 [Eq. (2)] was seen to provide the best agreement with the higher Re data and was used in the presented data. As may be seen, the extrapolations for the NACA 0015 and NACA 23012 sections show close accord with the experiment. However, agreement for the NACA 2412 is not as satisfactory. Looking at the experimental data curve shapes of the two Re numbers indicates that the curvatures are different, hence so are the slopes of the linearized polars. As a consequence, this case, presented to emphasize this point, violates the assumption of similar linearized polar slopes (which is usually the case, especially for wings).

Figure 4 presents data for two wings of 35 deg sweep as reported in [6] with Re variation from 2 to 10 million. The $AR = 5$ wing had a taper ratio of 0.7, whereas the $AR = 10$ wing had a taper ratio of 0.5. The experimental data indicated no observable shift in the zero lift drag coefficient; hence, this correction was not applied to the extrapolation. The agreement for the $AR = 5$ wing at high Re with the extrapolation is good, however, that of the $AR = 10$ wing is less satisfactory. It is evident that, for the $AR = 10$ wing, the stall region characteristics change when the Re is increased, a potential issue discussed earlier. As suggested by the chart in [4], the N65A012 section's stall characteristics change from being a leading-edge phenomenon ($Re = 2 \times 10^6$) to combined leading edge and trailing edge at $Re = 10 \times 10^6$.

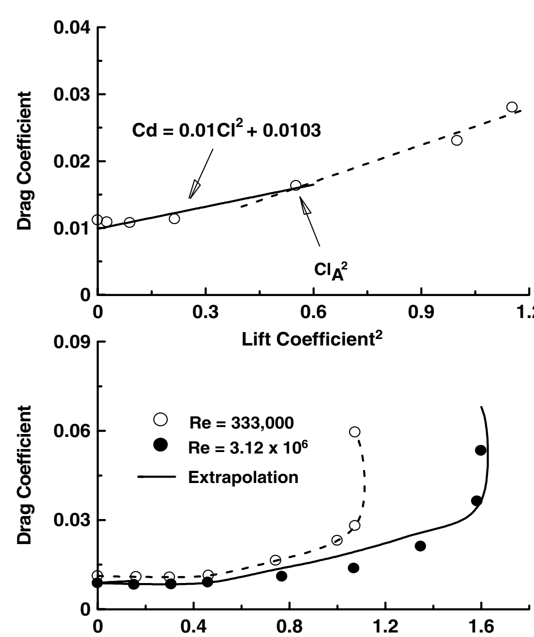

Fig. 3 Comparison of extrapolated polar with experiment, NACA 2412 section. Data from [5].

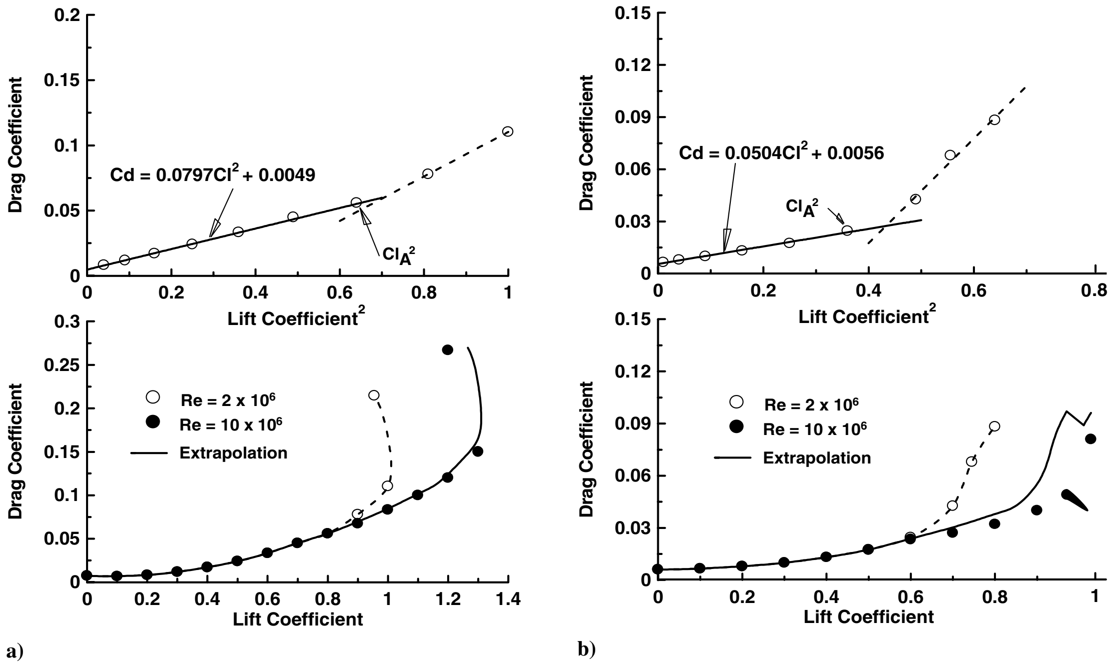

Fig. 4 Comparison of extrapolated polar with experiment, a) Æ = 5, N64A312 section and b) Æ = 10, N65A012 section. Data from [6].

Conclusions

A simple method is presented to extrapolate the drag polar from wind tunnel to flight Reynolds number. The methodology assumes that the nature of the stall and the slope of the linearized drag polar is preserved as the Re is increased. Cautions regarding these simplifying assumptions are given. Comparisons of the procedure with experimental data show encouraging accord. Cases are included to highlight instances where the method may lose validity due to violation of the assumptions.

Acknowledgments

The author would like to thank the Associate Editor and reviewer for their helpful comments and suggestions.

References

[1] Saltzman, E. J., and Ayers, T. G., "Review of Flight to Wind Tunnel Drag Correlations," *Journal of Aircraft*, Vol. 19, No. 10, 1982, pp. 801–811.
doi:10.2514/3.61561

[2] Petterson, K., and Rizzi, A., "Aerodynamic Scaling to Free Flight Conditions: Past and Present," *Progress in Aerospace Sciences*, Vol. 44, No. 4, 2008, pp. 295–313.
doi:10.1016/j.paerosci.2008.03.002

[3] Barlow, J. B., Rae, W., and Pope, A., *Low-Speed Wind Tunnel Testing*, 3rd ed., Wiley-Interscience, New York, 1999, pp. 367–425.

[4] Gault, D. E., "A Correlation of Low-Speed, Airfoil-Section Stalling Characteristics with Reynolds Number and Airfoil Geometry," NACA TN 3963, March 1957.

[5] Jacobs, E. N., and Sherman, A., "Airfoil Section Characteristics as Affected by Variations of the Reynolds Number," NACA Rept. 586, June 1936.

[6] Tinling, B., and Kolk, W. R., "The Effects of Mach Number and Reynolds Number on the Aerodynamic Characteristics of Several 12-Percent Thick Wings Having 35 deg of Sweepback and Various Amounts of Camber," NACA RM A50K27, Feb. 1951.